DIAS-STP-08-08

Constraints on an Asymptotic Safety Scenario for the Wess-Zumino Model

Oliver J. Rosten*
Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland

Using the nonrenormalization theorem and Pohlmeyer's theorem, it is proven that there cannot be an asymptotic safety scenario for the Wess-Zumino model unless there exists a non-trivial fixed point with (i) a negative anomalous dimension (ii) a relevant direction belonging to the Kähler potential.

PACS numbers: 11.10.Gh,11.10.Hi,11.30.Pb

In this note, we will consider the existence of certain renormalization group fixed points in theories of a chiral superfield. Suppose that a non-trivial fixed point exists and, moreover, that there is a renormalized trajectory [1] emanating from it, such that the low energy effective theory is well described by the Wess-Zumino model. It will be proven that, for such an asymptotic safety scenario [2] to occur, the putative fixed point must have both a negative anomalous dimension[1] and at least one relevant operator belonging to the Kähler potential. This generalizes earlier work [3] on zeros of the β-function of the Wess-Zumino model in a way that will be precisely spelt out below.

To formulate our argument, we introduce the Wilsonian effective action, S_Λ, constructed by integrating out degrees of freedom between the bare scale and a lower, effective scale, Λ (this implies that we have transferred to Euclidean space, so that momenta can be readily separated into large and small). The Wilsonian effective action, being infrared safe, does not suffer from the holomorphic anomaly in the massless case. Therefore, the nonrenormalization theorem always holds and the superpotential does not renormalize, even nonperturbatively [5].

To conveniently uncover fixed point behaviour, we rescale to dimensionless variables by dividing all quantities (coordinates and fields) by Λ raised to the appropriate scaling dimension. In the case of the chiral superfield, Φ, (and its conjugate) we must take account of the anomalous scaling according to

$$\Phi \to \Phi\sqrt{Z}\Lambda, \tag{1}$$

where Z is the field strength renormalization and the anomalous dimension is defined by

$$\gamma(\Lambda) \equiv \Lambda \frac{d\ln Z}{d\Lambda}. \tag{2}$$

As a consequence of the rescalings, the superpotential does now renormalize, but just according to the (anomalous) mass dimension of the various couplings. In particular, denoting the rescaled three-point superpotential by $\lambda(\Lambda)$, we have that

$$\beta_\lambda \equiv \Lambda \frac{d\lambda}{d\Lambda} = \frac{3\lambda\gamma}{2}. \tag{3}$$

In the rescaled variables, a fixed point is defined by

$$\Lambda\partial_\Lambda S_\star[\overline{\Phi}, \Phi] = 0, \tag{4}$$

where $\Lambda\partial_\Lambda$ is performed at constant $\overline{\Phi}, \Phi$ and a star is used to denote a fixed point quantity. Immediately, it is apparent from (3) and (4) that if $\lambda_\star \neq 0$, then it must be that $\gamma_\star = 0$. However, there is a theorem due to Pohlmeyer [6] which implies that, in the current scenario, the only scale invariant (i.e. fixed point) theory with $\gamma_\star = 0$ corresponds to the Gaussian fixed point. This was the reasoning used in [3] to rule out zeros of the β-function in the Wess-Zumino model; the same logic has also been applied to the O(N) symmetric Wess-Zumino model [7]. Here, though, we deal with general fixed point actions.

However, the condition that $\lambda_\star = 0$ is not sufficient to rule out an asymptotic safety scenario for the Wess-Zumino model. This is because, although a putative non-trivial fixed point cannot possess a three-point superpotential term, it could be that (i) λ constitutes a relevant direction at the fixed point (ii) trajectories initiated along the λ direction happen to flow towards the Gaussian fixed point. Note that a marginally relevant λ will not do, because this requires $\gamma_\star = 0$ and we again fall foul of Pohlmeyer's theorem.

Let us suppose that such a scenario is realized i.e. we perturb our fixed point action in the λ direction and flow towards the Gaussian fixed point. Now, in the vicinity of the Gaussian fixed point the low energy effective theory is described arbitrarily well by the Wess-Zumino model. This follows simply because, although λ is irrelevant with respect to the Gaussian fixed point, it is only marginally so, and so all other couplings (besides the mass, which can be ignored in this discussion) die off much faster.

Along the resulting renormalized trajectory between the two fixed points, we can write the action in 'self-similar' form [8, 9]. This means that all scale dependence of the action appears through $\lambda(\Lambda)$ which, as a

*Electronic address: orosten@stp.dias.ie

[1] It is worth pointing out that in the vicinity of a nonperturbative fixed point, we cannot rule out a negative anomalous dimension, γ, by the usual unitarity arguments. These relate the unitarity constraint $0 \leq Z \leq 1$ to a non-negative γ via a perturbative calculation; but there is no reason to believe such a calculation at a nonperturbative fixed point (see [4] for an interesting discussion on negative anomalous dimensions).

consequence of the nonrenormalization theorem, can be traded for the anomalous dimension:

$$S_\Lambda[\overline{\Phi}, \Phi] = S[\overline{\Phi}, \Phi](\gamma(\Lambda)). \qquad (5)$$

(More generally, a self-similar action depends on Λ through the relevant / marginally relevant couplings, as defined at the UV fixed point, and the anomalous dimension.) It is worth noting that self-similarity is a nonperturbative statement of renormalizability [9].

As just stated, in order for us to construct this renormalized trajectory, it must be that $\lambda(\Lambda)$ is relevant with respect to the non-trivial fixed point. This requires that $\gamma_\star < 0$, as follows from (3). Crucially, however, sufficiently close to the Gaussian fixed point—where we can rely on perturbation theory done with the Wess-Zumino model—we know that the anomalous dimension is positive.

Therefore, in going from the UV fixed point down to the vicinity of the Gaussian fixed point, $\gamma(\Lambda)$ must pass through zero (at least once). Consider the first time that this happens. Since all scale dependence along our renormalized trajectory is carried by $\gamma(\Lambda)$ then, if $\gamma(\Lambda)$ ever vanishes, we must be at a fixed point. Now, on the one hand, this fixed point cannot be the Gaussian one: the action in the vicinity of the Gaussian fixed point is (essentially) the Wess-Zumino action, but $\gamma(\Lambda)$ has not yet increased above zero, by assumption. On the other hand, Pohlmeyer's theorem tells us that this fixed point cannot be anything else! Therefore, our original assumption that there exists a non-trivial fixed point with a trajectory, spawned along the λ direction, emanating from it such that the low energy effective theory is well described by the Wess-Zumino model, must be incorrect.

However, suppose that the fixed point also possesses a relevant operator coming from the Kähler potential, $\mathcal{O}[\overline{\Phi}, \Phi]$, with coupling $g(\Lambda)$ (obviously, we can generalize this to several such operators). Perturbing the fixed point action in both the λ and g directions, the action along the resulting renormalized trajectory now reads

$$S_\Lambda[\overline{\Phi}, \Phi] = S[\overline{\Phi}, \Phi](g(\Lambda), \gamma(\Lambda)). \qquad (6)$$

Whilst it is still true that, in order for an asymptotic safety scenario to be realized for the Wess-Zumino model, the anomalous dimension must pass through zero, it is no longer true that the vanishing of $\gamma(\Lambda)$ at some scale necessarily corresponds to fixed point, since $g(\Lambda)$ could still be flowing.

Assuming such an asymptotic safety scenario to exist, we now have the following picture of the renormalization group flows. If we perturb away from the non-trivial fixed point in just the λ direction, then we must shoot off away from the Gaussian fixed point. (A finite distance along the resulting trajectory, it may be that $\mathcal{O}[\overline{\Phi}, \Phi]$ is generated, but now we have $g(\Lambda) = g(\gamma(\Lambda))$.) However, by perturbing the fixed point in both the λ and g directions, we flow towards the Gaussian fixed point, with the low energy effective action being well described by the Wess-Zumino action. The question as to whether such non-trivial fixed points actually exist will be addressed in a companion paper [10].

Acknowledgments

I acknowledge IRCSET for financial support. I would like to thank Denjoe O'Connor for comments on the manuscript and, particularly, Werner Nahm for a very useful discussion.

[1] K. G. Wilson and J. Kogut, "The Renormalization group and the epsilon expansion," *Phys. Rept.* **12** (1974) 75.
[2] S. Weinberg, "Ultraviolet Divergences in Quantum Theories of Gravitation," in *General Relativity*, S. W. Hawking and W. Israel, eds., p. 790. CUP, 1979.
[3] S. Ferrara, J. Wess, and B. Zumino, "Supergauge Multiplets and the Gell-Mann–Low Eigenvalue," *Nucl. Phys.* **B 77** (1974) 413.
[4] K. Higashijima and E. Itou, "Unitarity bound of the wave function renormalization constant," *Prog. Theor. Phys.* **110** (2003) 107, hep-th/0304047.
[5] N. Seiberg, "Naturalness versus supersymmetric nonrenormalization theorems," *Phys. Lett.* **B 318** (1993) 469, hep-ph/9309335.
[6] K. Pohlmeyer, "The Jost-Schroer Theorem for Zero-Mass Fields," *Commun. Math. Phys.* **12** (1969) 204.
[7] C. R. Nappi, "On O(N) Symmetric Wess-Zumino Type Models," *Phys. Rev.* **D 28** (1983) 3090.
[8] D. V. Shirkov, "The Renormalization Group Method and Functional Self-Similarity in Physics," *Theor. Math. Phys.* **60** (1985) 778 [Teor. Mat. Fiz. **60** (1984) 218].
[9] T. R. Morris, "Elements of the continuous renormalization group," *Prog. Theor. Phys.* **131** (1998) 395, hep-th/9802039.
[10] O. J. Rosten, "Work in Progress."